Supplemental Appendix

Derivation of the improved Adrogué-Madias (A-M) equation

See main text under the heading "Savory Math" for details on the variables. The general strategy uses an accounting of the total body cationic osmoles (TBCO), the relevant ones being Na^+ and K^+ for dysnatremia. When an IV fluid is infused, its sodium and potassium amounts are added to the numerator of the Edelman fraction, and its volume is added to the denominator. This gives the theoretical new serum sodium, or $[\mathrm{Na}]_2$. Subtract the initial serum sodium, or $[\mathrm{Na}]_1$, to get:

$$\Delta[\mathrm{Na}] = [\mathrm{Na}]_2 - [\mathrm{Na}]_1 = \frac{\mathrm{Na_e} + \mathrm{K_e} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V}{\mathrm{TBW} + V} - \frac{\mathrm{Na_e} + \mathrm{K_e}}{\mathrm{TBW}} \tag{4}$$

Then manipulate equation (4), following the rules of algebra:

$$\Delta[\mathrm{Na}] = \frac{(\mathrm{Na_e} + \mathrm{K_e} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V) \cdot \mathrm{TBW}}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}} - \frac{(\mathrm{Na_e} + \mathrm{K_e}) \cdot (\mathrm{TBW} + V)}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}}$$

$$\Delta[\mathrm{Na}] = \frac{\mathrm{Na_e} \cdot \mathrm{TBW} + \mathrm{K_e} \cdot \mathrm{TBW} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V \cdot \mathrm{TBW}}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}} - \frac{\mathrm{Na_e} \cdot \mathrm{TBW} + \mathrm{Na_e} \cdot V + \mathrm{K_e} \cdot \mathrm{TBW} + \mathrm{K_e} \cdot V}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}}$$

$$\Delta[\mathrm{Na}] = \frac{\mathrm{Na_e} \cdot \mathrm{TBW} - \mathrm{Na_e} \cdot \mathrm{TBW} + \mathrm{K_e} \cdot \mathrm{TBW} - \mathrm{K_e} \cdot \mathrm{TBW} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V \cdot \mathrm{TBW} - \mathrm{Na_e} \cdot V - \mathrm{K_e} \cdot V}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}}$$

$$\Delta[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V \cdot \mathrm{TBW} - \mathrm{Na_e} \cdot V - \mathrm{K_e} \cdot V}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}}$$

$$\Delta[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V \cdot \mathrm{TBW}}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}} - \frac{\mathrm{Na_e} \cdot V + \mathrm{K_e} \cdot V}{(\mathrm{TBW} + V) \cdot \mathrm{TBW}}$$

$$\Delta[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V}{\mathrm{TBW} + V} - \underbrace{\frac{\mathrm{Na_e} + \mathrm{K_e}}{\mathrm{TBW}}}_{\text{Edelman}} \cdot \frac{V}{\mathrm{TBW} + V}$$

Abridge $[\mathrm{Na}]_1$ to $[\mathrm{Na}]$, i.e., the patient's initial serum sodium, and substitute in $[\mathrm{Na}] = \frac{\mathrm{Na_e} + \mathrm{K_e}}{\mathrm{TBW}}$:

$$\Delta[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V}{\mathrm{TBW} + V} - [\mathrm{Na}] \cdot \frac{V}{\mathrm{TBW} + V}$$

$$\Delta[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V - [\mathrm{Na}] \cdot V}{\mathrm{TBW} + V}$$

$$\Delta[\text{Na}] = \frac{[\text{Na+K}]_{\text{IVF}} - [\text{Na}]}{\text{TBW} + \underbrace{V}_{\substack{\text{any}\\\text{vol.}}}} \underbrace{\cdot V}_{\substack{\text{scaling}\\\text{step}}} \tag{5}$$

Equation (5) is the improved version of the A-M formula. It accommodates any volume of infusate (V), not just one liter, and includes the scaling step (multiply by V).

Limit calculation proves that the correct scaling prohibits $\Delta[\text{Na}]$ from going to infinity:

Using just the right-hand side of the improved A-M equation (5), which is the delta [Na], we can ask how the serum sodium will change if an extremely large volume of IV fluid is infused. That is mathematically equivalent to letting the V go to infinity:

$$\lim_{V\to\infty} \frac{[\text{Na+K}]_{\text{IVF}} - [\text{Na}]}{\text{TBW} + V} \cdot V$$

$$\lim_{V\to\infty} \frac{[\text{Na+K}]_{\text{IVF}} - [\text{Na}]}{\cancel{\text{TBW} + V}^{\infty}} \cdot \cancel{V}^{\infty}$$

$$\lim_{V\to\infty} [\text{Na+K}]_{\text{IVF}} - [\text{Na}]$$

$$\lim_{V\to\infty} 154 - 111 = 43$$

The last line using example values for $[\text{Na+K}]_{\text{IVF}}$ and [Na] comes from the hypothetical in the main text about infusing normal saline (NS) into a patient whose serum sodium is 111 mEq/L. No matter how much NS is given, the delta [Na] will max out at 43 mEq/L.

Flip the equation: enter a desired delta [Na] and get the IV fluid volume to infuse

Rearrange equation (5) to solve not for delta [Na] but for V:

$$\Delta\,[\mathrm{Na}] = \frac{[\mathrm{Na{+}K}]_{\mathrm{IVF}} - [\mathrm{Na}]}{\mathrm{TBW} + V} \cdot V$$

$$\begin{aligned}
\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW} + \Delta\,[\mathrm{Na}] \cdot V &= [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V - [\mathrm{Na}] \cdot V \\
\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW} &= [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V - ([\mathrm{Na}] \cdot V + \Delta\,[\mathrm{Na}] \cdot V) \\
\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW} &= [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V - ([\mathrm{Na}] + \Delta\,[\mathrm{Na}]) \cdot V
\end{aligned}$$

Substitute in $[\mathrm{Na}] + \Delta[\mathrm{Na}] = [\mathrm{Na}]_2$, where $[\mathrm{Na}]_2$ is the target serum sodium:

$$\begin{aligned}
\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW} &= [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V - [\mathrm{Na}]_2 \cdot V \\
\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW} &= ([\mathrm{Na{+}K}]_{\mathrm{IVF}} - [\mathrm{Na}]_2) \cdot V
\end{aligned}$$

$$V = \frac{\Delta\,[\mathrm{Na}] \cdot \mathrm{TBW}}{[\mathrm{Na{+}K}]_{\mathrm{IVF}} - [\mathrm{Na}]_2} \qquad (6)$$

Equation (6) is more useful to the clinician who often has a desired delta [Na] in mind and wants to know how to achieve it in terms of an IV fluid volume to prescribe.

Slope and y-intercept

Edelman et al.'s scatter plot (their Figure 7) suggested that the relationship between [Na] and $\frac{\mathrm{Na_e + K_e}}{\mathrm{TBW}}$ was a diagonal line. However, that best-fit line did not have a slope of 1 (rather, 1.11) and did not have a y-intercept of 0 (rather, -25.6). This skewing and displacement of the line are believed to be caused by a Gibbs-Donnan equilibrium. Thus, the Edelman equation without simplification is:

$$[\mathrm{Na}] = m \cdot \frac{\mathrm{Na_e + K_e}}{\mathrm{TBW}} + b$$

where m is the slope (that is close to but not exactly 1) and b is the y-intercept (that is close to but not exactly 0). Using the full Edelman equation above to re-derive the delta [Na] equations may improve their accuracy. First, we re-derived the Nguyen-Kurtz equation because it is the most comprehensive and versatile; it includes a generic input, a generic output, and a therapeutic

IV fluid. Then the simpler sodium equations, which are just special cases, flow directly from Nguyen-Kurtz in its slope/y-intercept format. For example, A-M uses only the therapeutic IV fluid, so zeroing out the generic input and generic output in Nguyen-Kurtz will yield the A-M equation in *its* own slope/y-intercept format. The same principle applies to Barsoum-Levine.

By the full Edelman equation, the starting sodium is $[\mathrm{Na}]_1 = m \cdot \frac{\mathrm{Na_e+K_e}}{\mathrm{TBW}} + b$. As before, the therapeutic IV fluid's relevant parameters can be denoted as $[\mathrm{Na{+}K}]_{\mathrm{IVF}}$ and V_{IVF}. The generic input variables can be called $[\mathrm{Na{+}K}]_{\mathrm{In}}$ and V_{In}, and the generic output variables can be called $[\mathrm{Na{+}K}]_{\mathrm{Out}}$ and V_{Out}. When the effects of all these gains/losses are entered into the full Edelman equation, the ending sodium is:

$$[\mathrm{Na}]_2 = m \cdot \frac{\mathrm{Na_e} + \mathrm{K_e} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V_{\mathrm{IVF}} + [\mathrm{Na{+}K}]_{\mathrm{In}} \cdot V_{\mathrm{In}} - [\mathrm{Na{+}K}]_{\mathrm{Out}} \cdot V_{\mathrm{Out}}}{\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}}} + b$$

Then the delta [Na] is calculated by:

$$\Delta[\mathrm{Na}] = [\mathrm{Na}]_2 - [\mathrm{Na}]_1 = m \cdot \frac{\mathrm{Na_e} + \mathrm{K_e} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V_{\mathrm{IVF}} + [\mathrm{Na{+}K}]_{\mathrm{In}} \cdot V_{\mathrm{In}} - [\mathrm{Na{+}K}]_{\mathrm{Out}} \cdot V_{\mathrm{Out}}}{\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}}} + b - \left(m \cdot \frac{\mathrm{Na_e} + \mathrm{K_e}}{\mathrm{TBW}} + b\right)$$

First, the b's are subtracted out. Next, factor out the m's:

$$\Delta[\mathrm{Na}] = m \cdot \left[\frac{\mathrm{Na_e} + \mathrm{K_e} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V_{\mathrm{IVF}} + [\mathrm{Na{+}K}]_{\mathrm{In}} \cdot V_{\mathrm{In}} - [\mathrm{Na{+}K}]_{\mathrm{Out}} \cdot V_{\mathrm{Out}}}{\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}}} - \frac{\mathrm{Na_e} + \mathrm{K_e}}{\mathrm{TBW}}\right]$$

Put what is inside the brackets over a common denominator:

$$\Delta[\mathrm{Na}] = m \cdot \left[\frac{(\mathrm{Na_e} + \mathrm{K_e}) \cdot \mathrm{TBW} + [\mathrm{Na{+}K}]_{\mathrm{IVF}} \cdot V_{\mathrm{IVF}} \cdot \mathrm{TBW} + [\mathrm{Na{+}K}]_{\mathrm{In}} \cdot V_{\mathrm{In}} \cdot \mathrm{TBW} - [\mathrm{Na{+}K}]_{\mathrm{Out}} \cdot V_{\mathrm{Out}} \cdot \mathrm{TBW}}{(\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}}) \cdot \mathrm{TBW}} - \frac{(\mathrm{Na_e} + \mathrm{K_e}) \cdot (\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}})}{(\mathrm{TBW} + V_{\mathrm{IVF}} + V_{\mathrm{In}} - V_{\mathrm{Out}}) \cdot \mathrm{TBW}}\right]$$

The $(\mathrm{Na_e} + \mathrm{K_e}) \cdot \mathrm{TBW}$ terms are subtracted out, leaving:

$$\Delta[\text{Na}] = m \cdot \left[\frac{[\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} \cdot \text{TBW} + [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} \cdot \text{TBW} - [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}} \cdot \text{TBW}}{(\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}) \cdot \text{TBW}} - \frac{(\text{Na}_\text{e} + \text{K}_\text{e}) \cdot (V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}})}{(\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}) \cdot \text{TBW}}\right]$$

Above, in the first fraction, the TBW's cancel out, leaving:

$$\Delta[\text{Na}] = m \cdot \left[\frac{[\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} + [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} - [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} - \frac{(\text{Na}_\text{e} + \text{K}_\text{e}) \cdot (V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}})}{(\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}) \cdot \text{TBW}}\right]$$

Above, in the second fraction, isolate the $\frac{\text{Na}_\text{e}+\text{K}_\text{e}}{\text{TBW}}$ into a separate multiplier:

$$\Delta[\text{Na}] = m \cdot \left[\frac{[\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} + [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} - [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} - \frac{V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} \cdot \frac{\text{Na}_\text{e} + \text{K}_\text{e}}{\text{TBW}}\right]$$

The $\frac{\text{Na}_\text{e}+\text{K}_\text{e}}{\text{TBW}}$ is equal to $\frac{[\text{Na}]_1 - b}{m}$, seen by rearranging $[\text{Na}]_1 = m \cdot \frac{\text{Na}_\text{e}+\text{K}_\text{e}}{\text{TBW}} + b$. Substituting in:

$$\Delta[\text{Na}] = m \cdot \left[\frac{[\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} + [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} - [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} - \frac{V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} \cdot \frac{[\text{Na}]_1 - b}{m}\right]$$

Distribute the m and combine the fractions over a common denominator:

$$\Delta[\text{Na}] = \frac{m \cdot [\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} + m \cdot [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} - m \cdot [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}} - (V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}) \cdot ([\text{Na}]_1 - b)}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}}$$

Group the similar terms according to their volume multipliers:

$$\Delta[\text{Na}] = \frac{m \cdot [\text{Na+K}]_{\text{IVF}} \cdot V_{\text{IVF}} + b \cdot V_{\text{IVF}} - [\text{Na}]_1 \cdot V_{\text{IVF}} + m \cdot [\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} + b \cdot V_{\text{In}} - [\text{Na}]_1 \cdot V_{\text{In}} - m \cdot [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}} - b \cdot V_{\text{Out}} + [\text{Na}]_1 \cdot V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}}$$

Factor out the volume multipliers:

$$\Delta[\text{Na}] = \frac{(m \cdot [\text{Na+K}]_{\text{IVF}} + b - [\text{Na}]_1) \cdot V_{\text{IVF}} + (m \cdot [\text{Na+K}]_{\text{In}} + b - [\text{Na}]_1) \cdot V_{\text{In}} - (m \cdot [\text{Na+K}]_{\text{Out}} + b - [\text{Na}]_1) \cdot V_{\text{Out}}}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}}$$

Break up the one big fraction into three smaller fractions:

$$\Delta[\text{Na}] = \underbrace{\frac{m \cdot [\text{Na+K}]_{\text{IVF}} + b - [\text{Na}]_1}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} \cdot V_{\text{IVF}}}_{\text{Module for therapeutic IV fluid}} + \underbrace{\frac{m \cdot [\text{Na+K}]_{\text{In}} + b - [\text{Na}]_1}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} \cdot V_{\text{In}}}_{\text{Module for generic input}} - \underbrace{\frac{m \cdot [\text{Na+K}]_{\text{Out}} + b - [\text{Na}]_1}{\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}} \cdot V_{\text{Out}}}_{\text{Module for generic output}} \quad (7)$$

Each of the smaller fractions is seen to function like a module for a particular input/output. The pattern is consistently repeated and can be generalized into a rubric for incorporating any number of gains and losses:

1) Sign: Inputs (including the therapeutic IV fluid) are added, and outputs are subtracted.

2) Each input or output fluid's [Na+K] is modified by the slope and y-intercept, as in $m \cdot [\text{Na+K}]_{\text{IVF}} + b$ for example, similar to the way that the $\frac{\text{Na}_e+\text{K}_e}{\text{TBW}}$ is modified by the slope and y-intercept in the full Edelman equation.

3) From part 2), subtract the patient's starting sodium, $[\text{Na}]_1$. This forms the numerator.

4) Divide by a denominator that is basically the new body volume after all gains and losses: $\text{TBW} + V_{\text{IVF}} + V_{\text{In}} - V_{\text{Out}}$.

5) Multiply the whole fraction by the volume of the input or output, V_{IVF} for example.

6) Sum up all of the modules to calculate the delta [Na].

Knowing the rubric, one can fathom the complexity of the equation and even reconstruct it.

Equation (7) is the delta [Na] version of the Nguyen-Kurtz sodium formula with a slope and y-intercept. From this all-inclusive version, modules can be omitted to quickly arrive at the other eponymous sodium equations in their slope and y-intercept forms. For example, the Adrogué-Madias formula considers only the IV fluid, so removing the generic input and generic output reduces equation (7) to:

$$\Delta[\text{Na}] = \frac{m \cdot [\text{Na+K}]_{\text{IVF}} + b - [\text{Na}]_1}{\text{TBW} + V_{\text{IVF}}} \cdot V_{\text{IVF}} \tag{8}$$

This A-M formula with a slope and y-intercept is reminiscent of equation (5), reproduced here:

$$\Delta[\text{Na}] = \frac{[\text{Na+K}]_{\text{IVF}} - [\text{Na}]}{\text{TBW} + V} \cdot V$$

If $m = 1$ and $b = 0$, as A-M assumes, then equation (8) turns into equation (5) *exactly*, as it should. This internal consistency also serves as a check on all of the preceding algebra.

Flip the equation again: slope and y-intercept edition

As with flipping equation (5)—delta [Na]—into equation (6)—volume of IVF—, equation (7) can also be rearranged to solve for the volume of therapeutic IV fluid to infuse. Arguably, this is the more useful information to know when treating a dysnatremia. Without delving into the algebraic details, we present the formula for the IVF volume:

$$V_{\text{IVF}} = \frac{\Delta[\text{Na}] \cdot \text{TBW} - \overbrace{(m \cdot [\text{Na+K}]_{\text{In}} + b - [\text{Na}]_2) \cdot V_{\text{In}}}^{\text{Module for generic input}} + \overbrace{(m \cdot [\text{Na+K}]_{\text{Out}} + b - [\text{Na}]_2) \cdot V_{\text{Out}}}^{\text{Module for generic output}} \mp \cdots}{m \cdot [\text{Na+K}]_{\text{IVF}} + b - [\text{Na}]_2} \tag{9}$$

Equation (9) is a little harder to remember, but a few rules will help in reconstructing it:

1) <u>Sign</u>: Contrarily, inputs are subtracted, and outputs are added. The $\mp$ indicates the opposite of the expected sign, and the ellipsis indicates the modular, expandable nature of the equation.
2) The numerator starts off with the $\Delta[\text{Na}] \cdot \text{TBW}$, like in equation (6). This gives a sense of how much cationic solute should be gained or lost to achieve a desired change in serum sodium.
3) From there, the numerator accommodates as many modules as there are inputs and outputs.
4) Each module is stereotypically an I/O fluid's [Na+K], modified by the slope and y-intercept, minus the *ending* serum sodium, $[\text{Na}]_2$. Multiply that difference by the volume of the I/O.
5) Divide the entire numerator by a denominator consisting of the therapeutic IV fluid's [Na+K], modified by the slope and y-intercept, minus the $[\text{Na}]_2$.

<u>Check</u>: If generic I's/O's are zeroed out, equation (9) turns into equation (6) for $m = 1, b = 0$:

$$V = \frac{\Delta[\text{Na}] \cdot \text{TBW}}{[\text{Na+K}]_{\text{IVF}} - [\text{Na}]_2}$$

Amazingly, our equation (9) is mathematically identical to the Nguyen-Kurtz equation (6) from their article (*Clin Exp Nephrol* 7: 125-137, 2003). Written in our nomenclature, their (6) was:

$$V_{\text{IVF}} = \frac{([\text{Na}]_1 - b) \cdot \text{TBW} - ([\text{Na}]_2 - b) \cdot (\text{TBW} + V_{\text{In}} - V_{\text{Out}}) + m \cdot ([\text{Na+K}]_{\text{In}} \cdot V_{\text{In}} - [\text{Na+K}]_{\text{Out}} \cdot V_{\text{Out}})}{[\text{Na}]_2 - b - m \cdot [\text{Na+K}]_{\text{IVF}}}$$

That can be massaged into our equation (9), which has a more memorable pattern and grouping.